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The intensity of a single-mode gas laser as a function of cavity Q

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Abstract. The single-mode intensity of a 0·6328 μm He–Ne laser has been measured as a function of the cavity Q. Significant deviations have been found from the predictions of the Lamb semi-classical theory, particularly near threshold. The results have been fully explained using the quantum theory of Scully and Lamb.

1. Introduction

In experiments to investigate the Lamb dip (see, for example, McFarlane *et al.* 1963), the intensity of a gas laser operating in a single mode has been measured as a function of the detuning of the mode from the line centre. The intensity has not previously been measured as a function of cavity Q because of the difficulty of making sufficiently small changes in Q which can be accurately determined. The skew Brewster angle flat (Allen *et al.* 1968), however, allows measurements of this kind to be made.

2. Experimental details

A He–Ne laser tube of 2 mm bore and 30 cm length was mounted in a plano-concave cavity of 50 cm overall length. A skew Brewster angle flat, mounted as described elsewhere and driven by a constant speed motor, was placed in the cavity at the plane mirror end. The laser beam from this end of the cavity was focused on the entrance slit of a monochromator. The entrance and exit slits were opened to full width and the exit beam was checked to ensure that all the light reached the photomultiplier detector. The detector output was coupled through a potentiometer to a chart recorder. Voltage calibration was achieved using a valve voltmeter. Motor speeds were chosen so that one inch of chart paper corresponded to 0·9° rotation of the skew Brewster angle flat; this was equivalent to a change in inserted cavity loss of 0·01% for small inserted losses.

The inter-mode beats could be monitored using a r.f. spectrum analyser system as described by Allen *et al.* (1969), so that it was possible to know when the laser was working in a single mode. The laser discharge current was stabilized and the cavity shielded from thermal currents. The whole apparatus was mounted on a brick pier to minimize the effects of vibration.

3. Results

The cavity mirrors were tuned so that at the chosen value of tube operating current (typically in the region of 3 mA) the only mode able to oscillate was a single TEM_{00q} mode.

Typical plots of intensity against inserted loss obtained from this experiment are shown in figure 1. There are two important features to notice: the curved region well above threshold and the long tail near threshold.

4. Theory

The Lamb (1964) semi-classical theory of the gas laser predicts that the intensity of a single mode is, in a steady state,

$$I = \frac{-\alpha_Q + \alpha_G}{\beta} \tag{1}$$

where α_Q depends inversely on Q, α_G describes the gain of the active medium and β is a saturation parameter. For a constant detuning of the mode from the line centre, α_G and β are constant. Since $Q = k/L$ (Fox and Li 1961) I should be a linear function of cavity loss.

The quantum theory of the laser due to Scully and Lamb (1968) provides a description which includes the effects of spontaneous emission in a rigorous fashion, and might therefore be expected to give better agreement with the experimental results, particularly near threshold. Scully and Lamb develop expressions for the expectation value of the number

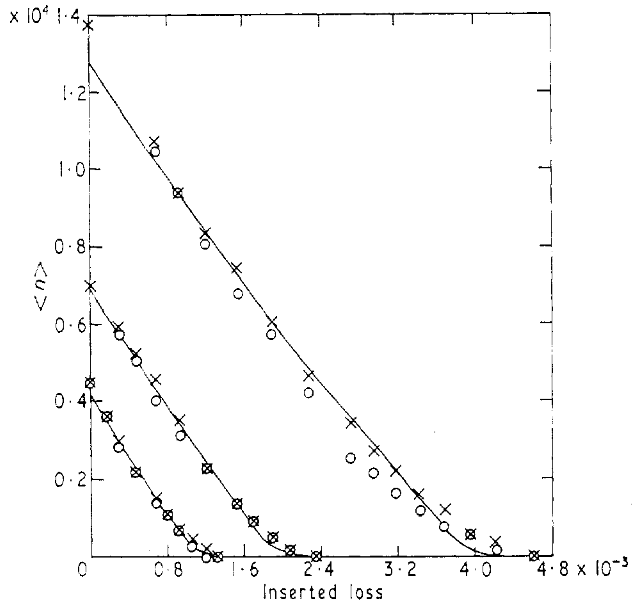

Figure 1. Single-mode intensity as a function of inserted loss; theoretical curves and experimental points for three values of $\langle n \rangle_p$. For each experimental run two measurements of intensity can be made for each value of loss. These are shown by circles and crosses.

of photons in a mode in the cavity, $\langle n \rangle$, for the case of a single mode with zero detuning, and in the absence of atomic motion. Consequently the quantum mechanical analogue of I may be considered to be $\langle n \rangle$. The quantum theory gives an expression for $\langle n \rangle$:

$$\langle n \rangle = \frac{A}{B}\,\frac{A-C}{C} + \frac{A}{B}\,\rho_{00}. \tag{2}$$

ρ_{00} may be thought of as a term representing spontaneous emission which has been amplified by the active medium. A is the gain of the lasing medium, C is the cavity loss parameter, B is a non-linear saturation parameter and

$$\rho_{00} = \frac{Z^{-1}(A^2/BC)^{A/B}}{(A/B)!}.$$

where

$$Z = \sum_{n=0}^{\infty} \frac{(A^2/BC)^{n+A/B}}{(n+A/B)!}.$$

This can be written as

$$\rho_{00} = \left\{1 + \frac{A^2/BC}{1+A/B} + \frac{(A^2/BC)^2}{(1+A/B)(2+A/B)} + \dots\right\}^{-1}. \tag{3}$$

Since above threshold $A/B > 1$, it is convenient to rewrite equation (3) in the form

$$\rho_{00} = \left\{1 + \frac{A/C}{B/A+1} + \frac{(A/C)^2}{(B/A+1)(2B/A+1)} + \ldots\right\}^{-1}$$
$$= \left[\sum_{n=0}^{\infty}\left\{\left(\frac{A}{C}\right)^n \Big/ \prod_{r=0}^{n}\left(1+\frac{rB}{A}\right)\right\}\right]^{-1}.$$

This term is clearly only important very near threshold, where A/C is very nearly unity.

The behaviour of equation (2) can therefore be considered in three separate regions. Far above threshold the complete first term must be used,

$$\langle n \rangle = \frac{A}{B}\left(\frac{A}{C} - 1\right).$$

Nearer to threshold this can be approximated to $\langle n \rangle = (A-C)/B$ which is the equivalent to the semi-classical case quoted in equation (1). Very close to threshold ρ_{00} becomes important and has the effect of adding a 'tail' to $\langle n \rangle$. If C is recognized to be equivalent to the term α_Q of the semi-classical theory, it becomes possible to express the important parameters A/B and A/C in terms of measurable quantities.

Let L be an inserted loss, L_c the residual cavity loss and L_m the maximum inserted loss. Then $C = k(L_c+L)$. Taking the semi-classical approximation $\langle n \rangle = (A-C)/B$, at threshold

$$A = C = k(L_c+L_m).$$

Above threshold

$$\langle n \rangle = \frac{A}{B}\left(\frac{A}{C} - 1\right)$$

and therefore for zero inserted loss the peak intensity can be written

$$\langle n \rangle_p = k(L_c+L_m)\left(\frac{L_c+L_m}{L_c} - 1\right) B^{-1}.$$

In principle, therefore, A/B and A/C are describable in terms of measureable quantities, i.e.

$$\frac{A}{B} = \frac{\langle n \rangle_p L_c}{L_m}, \qquad \frac{A}{C} = \frac{L_c+L_m}{L_c+L}.$$

The quantity $\langle n \rangle$ can be calculated from the photomultiplier output current if the assumption is made that the volume of the mode of the radiation field in the cavity is the same as the volume of the beam.

It was noted earlier that the experimental results did not have a long linear region above threshold, hence the first term in equation (2) must be used in full:

$$\langle n \rangle = \frac{A}{B}\left(\frac{A}{C} - 1\right) = \left(\frac{L_c+L_m}{L_c+L} - 1\right)\frac{L_c\langle n \rangle_p}{L_m}.$$

If a transformation $L' = (L_c+L)$ is made then,

$$\langle n \rangle = \left(\frac{L_c+L_m}{L'} - 1\right)\frac{L_c\langle n \rangle_p}{L_m}.$$

In this form $\langle n \rangle$ is a linear function of $1/L'$, with slope $(L_c+L_m)L_c\langle n \rangle_p/L_m$ and intercept on the $1/L'$ axis of $1/(L_c+L_m)$.

The experimental results plotted in this form give good straight lines except at threshold. Values of the slope and intercept determined from these lines have been used to calculate L_m and $\langle n \rangle_p$, and hence to find A/B and A/C. It should be noted that the intercept on the

$1/L'$ axis, $1/(L_c+L_m)$, determines the semi-classical threshold loss and not the actual experimental value, which is larger than that anticipated semi-classically owing to the effect of ρ_{00}.

The values obtained in this way for A/B and A/C have been used to evaluate equation (2) numerically on a digital computer. For the range of values of A/B used, the convergence of ρ_{00} is not a serious problem.

Figure 1 shows theoretical curves fitted by the method described above to three sets of experimental results. These curves correspond to different laser tube currents and, hence, to different values of maximum possible inserted loss. For each set of results two measurements of intensity can be made for a given value of inserted loss, corresponding to loss increasing and decreasing respectively.

5. Conclusions

It may be seen that good agreement is achieved between experiment and the quantum theory of the laser (Scully and Lamb 1968), even though the theory is for the case of zero atomic motion. Provided the time of experimental observation is such that the detuning does not change significantly (Jones *et al.* 1969), ignoring the effect of atomic motion would seem to be allowable, since the electromagnetic field interacts with the same atoms during the course of one experimental run.

It should be noted that the linear dependence of intensity upon the loss L, predicted by the semi-classical theory, is found to hold only over a small region. This occurs from slightly above threshold to a point considerably below maximum single-mode intensity. In terms of the relative excitation η defined by Lamb, the region of linearity is from $\eta = 1{\cdot}02 \pm 0{\cdot}01$ to $\eta = 1{\cdot}10 \pm 0{\cdot}05$.

It is not surprising that the semi-classical theory breaks down for high intensities, since it is based upon third-order perturbation theory. This assumes that in the equation determining the field amplitude, terms involving powers of I greater than the first are negligible.

The tail in the intensity curve for high inserted losses may be explained as the result of the purely quantum mechanical term ρ_{00}. This has the effect of increasing the value of the loss necessary for extinction above that predicted by semi-classical theory.

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